Test 2 - MTH 1400 Online

Dr. Graham-Squire, Summer 2016

8:58

Name: Key

I pledge that I have neither given nor received any unauthorized assistance on this exam.

____

(*signature*)

Need Ex. Cred.

DIRECTIONS

1. Don't panic.
2. Show all of your work and use correct notation! A correct answer with insufficient work or incorrect notation will lose points.
3. Clearly indicate your answer by putting a box around it.
4. Cell phones and computers are not allowed on this test. Calculators are allowed on the first 7 questions of the test, however you should still show all of your work. No calculators are allowed on the last 4 questions. If you change from the With calculator portion of the test to the No Calculator portion, it is fine to go back to the With Calculator portion again. However, once you turn in the No Calculator portion of the test, you CANNOT return to it.
5. Give all answers in exact form, not decimal form (that is, put π instead of 3.1415, $\sqrt{2}$ instead of 1.414, etc) unless otherwise stated.
6. If you need it, the quadratic formula is $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.
7. If you need it, the law of cosines is $c^2 = a^2 + b^2 - 2ab\cos(C)$.
8. Make sure you sign the pledge.
9. Number of questions = ~~11~~ 9. Total Points = ~~65~~ 60.

1. (6 points) If a car engine overheats, the resulting cooling process can be modeled by the equation

$$\ln\left(\frac{T-20}{180}\right) = -0.12t,$$

where T is the temperature of the engine t minutes after the car has overheated.

(a) Solve the equation for T. (that is, rewrite the equation in the form $T =$ _______, where _______ is an expression in terms of t). Show your work!

(b) Find the temperature of the engine 13 minutes after the car has overheated. Explain/show how you got your answer. Round answer to nearest 0.1 degrees

(a) $e^{\left(\ln\left(\frac{T-20}{180}\right)\right)} = e^{(-0.12t)}$

$180\left(\frac{T-20}{\cancel{180}}\right) = e^{-0.12t} \cdot 180$

$T - 20 = 180e^{-0.12t}$

$\boxed{T = 180e^{-0.12t} + 20}$

4

(b) $T = 180e^{-0.12(13)} + 20$

$T = 57.8^{\circ}$

2

2. (8 points) The current population (in 2016) of the US is 318.9 million. In 1900, the US population was 76.2 million. Assuming that the US population can be modeled by an exponential growth function (and that the population continues to grow at the same rate we have seen for the past 116 years), approximate when the US population will reach 1 billion (that is, 1,000 million). Make sure to show/explain your work.

$A = Pe^{rt}$ ✓

Let $t=0$ ~~be~~ 1900 0.5

$\Rightarrow P = 76.2$ 0.5

$$2016 - 1900 = 116$$

3

$$\Rightarrow \frac{318.9}{76.2} = \frac{\cancel{76.2}e^{r(116)}}{\cancel{76.2}}$$

+1 for knowing to solve for r

$$\ln\left(\frac{318.9}{76.2}\right) = \cancel{\ln} e^{116r}$$

$$\frac{\ln\left(\frac{318.9}{76.2}\right)}{116} = \frac{116r}{116}$$

$$r = \frac{\ln\left(\frac{318.9}{76.2}\right)}{116} \approx 0.1234$$

$\boxed{\text{In the year 2109 (or 2108) the U.S. population will reach 1 million}}$ 0.5

Need to find t such that $A = 1{,}000$

$$1{,}000 = 76.2e^{\left(\frac{\ln\left(\frac{318.9}{76.2}\right)}{116}\right)t}$$

2.5

$$\Rightarrow \ln\left(\frac{1{,}000}{76.2}\right) = \cancel{\ln e}^{\left(\frac{\ln\left(\frac{318.9}{76.2}\right)}{116}\right)t}$$

$$\frac{\ln\left(\frac{1{,}000}{76.2}\right)}{\left(\frac{\ln\left(\frac{318.9}{76.2}\right)}{116}\right)} = t$$

$\Rightarrow t = 208.6$ years

$\Rightarrow 1900 + 208.6 \approx 2109$

3. (6 points) (a) Use trigonometric relations to write the $\tan\theta$ solely in terms of $\cos\theta$, assuming that θ is in Quadrant III. So your answer should be in the form $\tan\theta =$ ______, where ______ is an expression in terms of $\cos\theta$. Make sure to show your work and/or explain your reasoning!

4

(b) Check to see if your answer is correct by evaluating both sides of the equation for $\theta = 4$ (Make sure your calculator is in radians!). Explain how you know that $\theta = 4$ is in Quadrant III.

(c)

(a) $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \sqrt{\sin^2\theta} = \sqrt{1-\cos^2\theta}$

$\sin\theta = \pm\sqrt{1-\cos^2\theta}$

Quad 3 $\Rightarrow$

$\sin\theta = -\sqrt{1-\cos^2\theta}$

$$\boxed{\tan\theta = \frac{-\sqrt{1-\cos^2\theta}}{\cos\theta}}$$

(b) $\tan 4 \stackrel{?}{=} \dfrac{-\sqrt{1-\cos^2 4}}{\cos 4}$

$1.1578 \stackrel{?}{=} \dfrac{-0.7568}{-0.6536}$

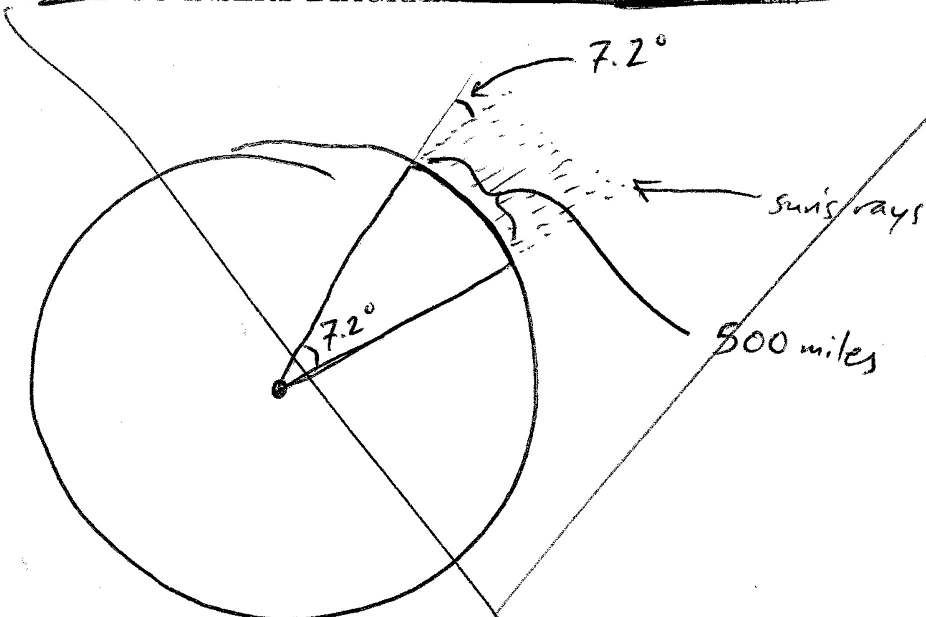
$1.1578 = 1.1578$

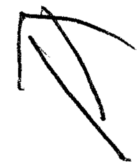
Know $\theta = 4$ is in Quad III b/c Quad 3 is between $\theta = \pi \approx 3.14$ and $\theta = \frac{3\pi}{2} \approx \frac{3(3.14)}{2} = 4.71$

and $3.14 < 4 < 4.71$

4. (4 points) The Greek mathematician Eratosthenes used a clever trick to estimate the circumference and radius of the earth. He noticed that on a certain day, the sun shone directly down a deep well in Syene. At that exact same time, in Alexandria (which is directly 500 miles north) the rays of the sun shone at an angle of 7.2° to the zenith. This indicated that the angle between the two (as measured from the center of the Earth, see diagram below) was also 7.2°. Use this information to calculate the circumference and radius of the Earth (Hint: you ~~will~~ may need to convert from degrees to radians at some point.)

~~NEED TO INSERT DIAGRAM LIKE ON [illegible]!!!!!~~





7.2° in radian is $(7.2^\circ)\cdot\left(\frac{\pi}{180^\circ}\right) = 0.04\pi$

so 0.04π radians = 500 miles. Circumference is

2π, so $\frac{2\pi}{0.04\pi}(0.04\pi) = 500$ miles $\left(\frac{2\pi}{0.04\pi}\right) = 500 \cdot 50$
$= 25{,}000$

$\Rightarrow$ circumference is $\approx 25{,}000$ $\Rightarrow$ radius is

$\frac{25{,}000}{2\pi} = \boxed{3978.87 \text{ miles}}$

~~Make 2 quest.~~ Make 2 quest.

5. (10 points) Julie is standing on top of a hill and looking at a flagpole of height h which is a distance of d feet away. Julie measures the angle of elevation to the top of the flagpole to be 18°, and the angle of depression to the bottom of the pole to be 14°.

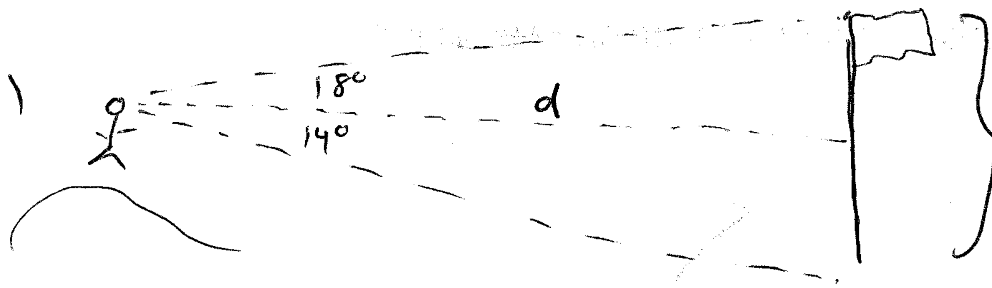
(a) diagram

(a) If Julie's distance d to the flagpole is 100 feet, calculate the height h of the flagpole.

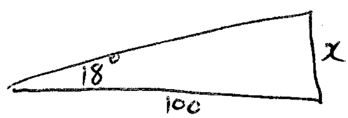
(b) ~~Now suppose~~ Julie does *not* know the distance d to the flagpole, ~~but she does know~~ that the flagpole is 60 feet ~~tall. Can she calculate~~ the distance d? If not, explain why not. ~~If so, calculate~~ it and explain how you got ~~your answer~~.

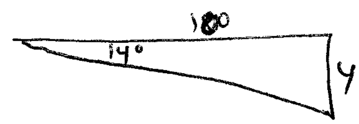
Round to nearest 0.01

(a)



(b)





$\tan 18° = \frac{x}{100}$

$x = 100 \cdot \tan(18°)$

$x = 32.49$ feet

$\tan 14° = \frac{y}{100}$

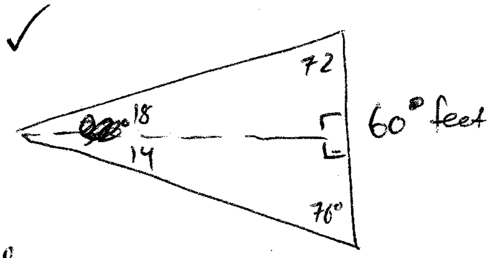
$y = 100(\tan 14°)$

$y = 24.93$

4

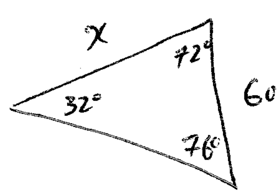
$\boxed{h = x + y = 57.42 \text{ feet}}$

(c) Yes, she can.



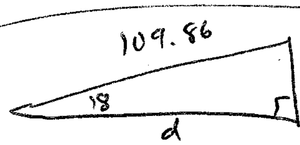
3

$\Rightarrow$



$\Rightarrow \frac{60}{\sin 32} = \frac{x}{\sin 76°}$ $\Rightarrow x = \sin 76° \left(\frac{60}{\sin 32}\right)$

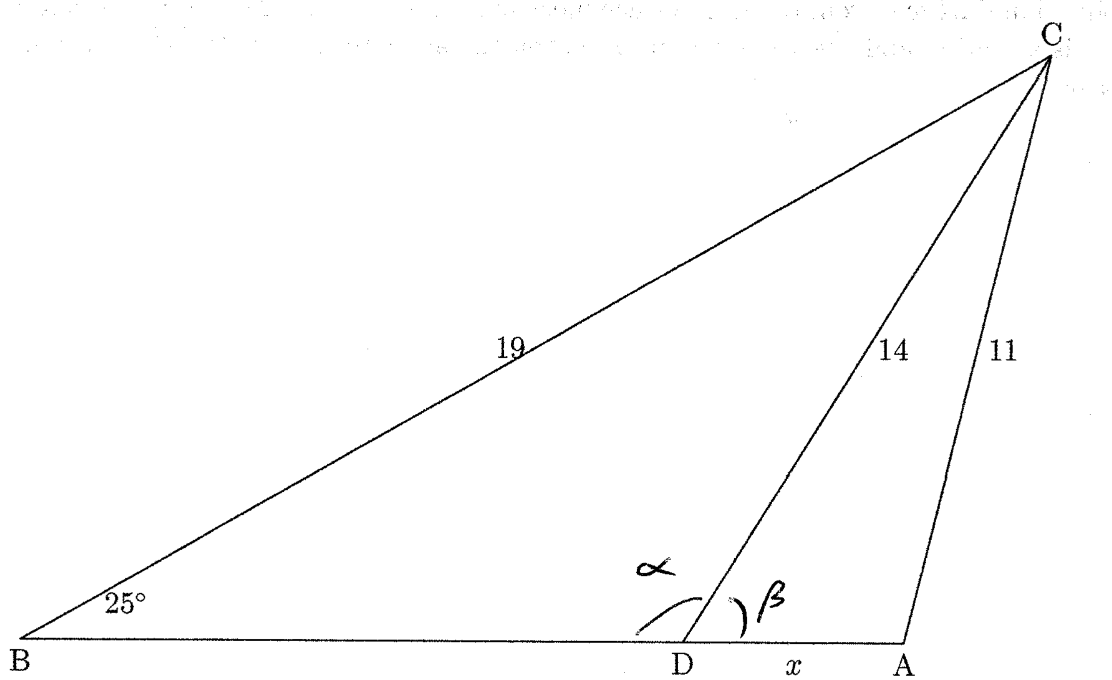
$x = 109.86$



$\cos 18° = \frac{d}{109.86}$

$\Rightarrow d = (109.86)(\cos 18°) = d = \boxed{104.48 \text{ fe}}$

6. (**6** points) In the diagram below, $\overline{BC} = 19$, $\overline{DC} = 14$, $\overline{AC} = 11$, angle $B = 25°$. Use laws of trigonometry to calculate the length of $x = \overline{AD}$. (Note: you can assume that $\overline{AB}$ is a straight line.) If there is more than one possible answer, you should judge from the diagram which answer you think is more correct, and explain your reasoning.



2.5

$$\frac{\sin \alpha}{19} = \frac{\sin 25°}{14} \Rightarrow \sin \alpha = 19\left(\frac{\sin 25°}{14}\right)$$

$$\alpha = \sin^{-1}\left(19\left(\frac{\sin 25°}{14}\right)\right)$$

$$\alpha = 35.00° \quad \text{or} \quad 180-35 = 145°$$

Since α looks obtuse, let $\alpha = 145°$ 0.5

$\Rightarrow \beta = 180° - 145° = 35°$

and use law of cosines:

2.5

$$11^2 = x^2 + 14^2 - 2(x)(14)\cos(35°)$$

$$\Rightarrow 0 = x^2 - 22.936x + 75$$

$$\Rightarrow x = \frac{22.936 \pm \sqrt{(22.936)^2 - 4(1)75}}{2(1)} = \frac{22.936 \pm 15.035}{2}$$

0.5 $= 18.99$ or 3.95 feet

Since x is smallest side of $\triangle ADC$, choose $\boxed{3.95\text{ ft}}$

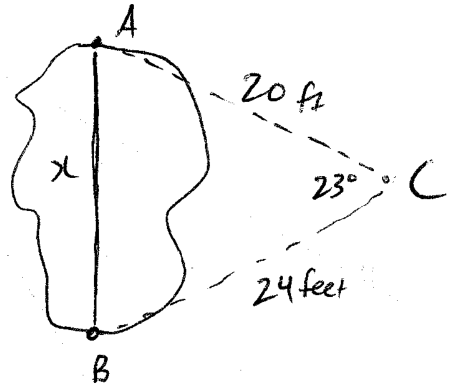
7. (6 points) Grace is a ranger at Yellowstone National Park, and she has to measure the size of a hot spring. Of course, the water in the spring is boiling hot, so she cannot measure length of the spring directly. Instead, she locates a point A at one end of the spring and a point B at the other end of the spring. From a point C that 20 feet from point A and 24 feet from point B, Grace measures angle C to be 23°. (a) Draw a diagram of this situation and (b) use the given information to calculate the length $\overline{AB}$ of the hot spring.

9:30 → 28

9:33

Round to nearest 0.01.

(a)



$$x^2 = 20^2 + 24^2 - 2(20)(24)\cos(23°)$$

$$x = \sqrt{20^2 + 24^2 - 2(20)(24)\cos(23°)}$$

$$\boxed{x = 9.61 \text{ feet}}$$

No Calculator Name: Key

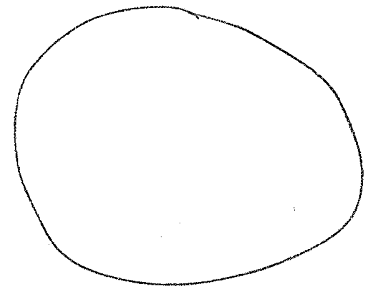
•Note that once you finish this portion of the test and turn it in, you CANNOT return to it!

(1.5 each

8. (12 points) Without using a calculator, calculate the following. If the expression does not exist or is undefined, say so and explain why. Make sure to show your work, if there is any work to be shown! If it would help you, you can fill out the unit circle on the next page.

1.5 each

(a) $\sin(60°) = \boxed{\frac{\sqrt{3}}{2}}$



(b) $\cos\left(\frac{7\pi}{6}\right) = -\cos\left(\frac{\pi}{6}\right) = \boxed{-\frac{\sqrt{3}}{2}}$

(c) $\tan\left(-\frac{3\pi}{4}\right) = \tan\left(\frac{5\pi}{4}\right) = \frac{\left(\frac{-\sqrt{2}}{2}\right)}{\left(-\frac{\sqrt{2}}{2}\right)} = \boxed{1}$

(d) $\sec(630°) = \sec(630-360) = \sec(270) = \frac{1}{\cos(270)} = \frac{1}{0} = \boxed{dne}$

cannot divide by zero!

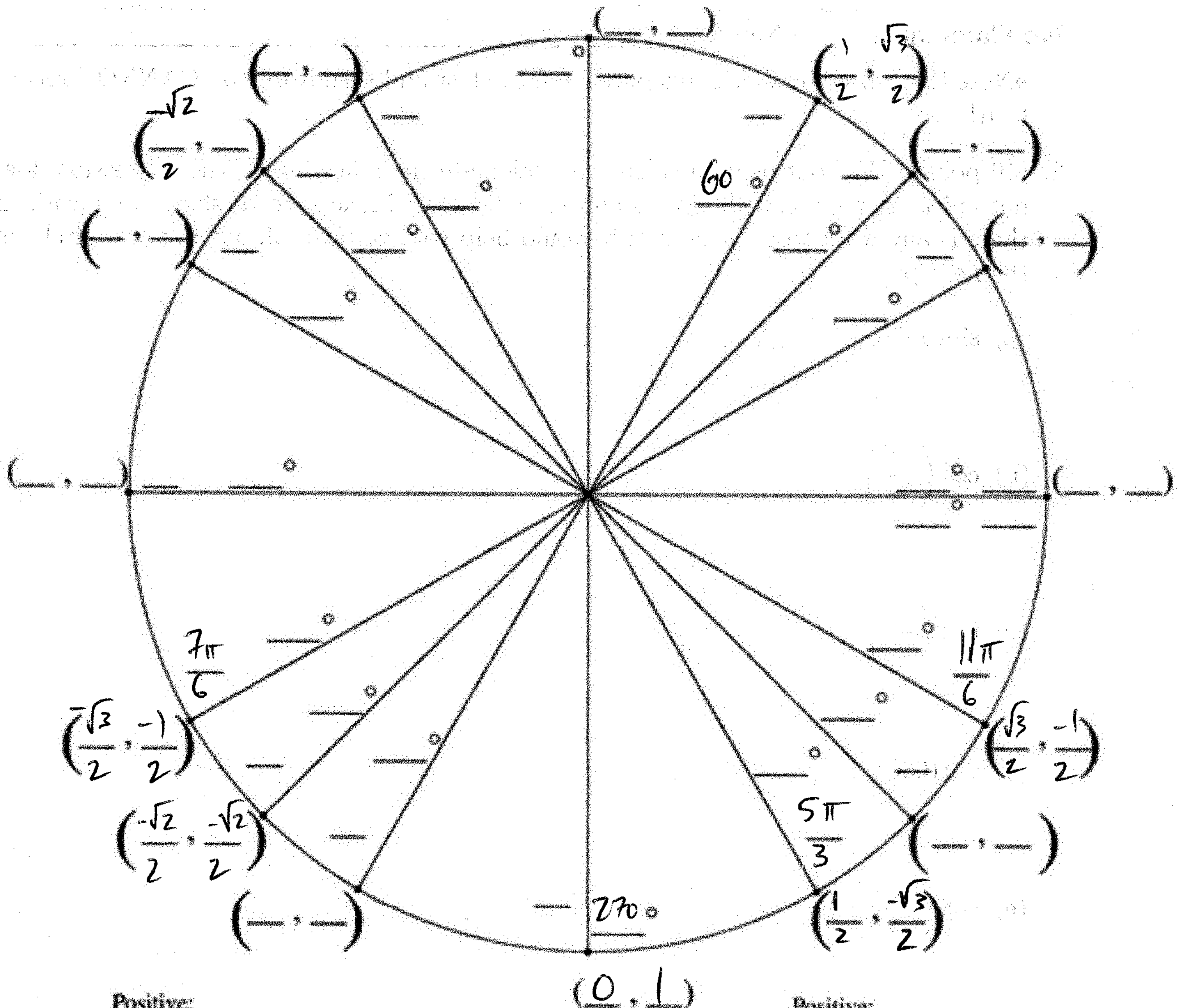
(e) $\cot\left(\frac{29\pi}{3}\right) = \cot\left(\frac{29\pi}{3} - \frac{24\pi}{3}\right) = \cot\left(\frac{5\pi}{3}\right) = \frac{\cos\left(\frac{5\pi}{3}\right)}{\sin\left(\frac{5\pi}{3}\right)} = \frac{\left(\frac{1}{2}\right)}{\left(-\frac{\sqrt{3}}{2}\right)} = \boxed{\frac{-1}{\sqrt{3}}}$

(f) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \boxed{\frac{3\pi}{4} \text{ or } 135°}$ (ref. angle is $\frac{\pi}{4}$, but must be in Quad II)

(g) $\arcsin(\sqrt{3}) = \boxed{\text{DNE}}$ b/c $\sqrt{3} > \sqrt{1} = 1$ and cannot have a sine value greater than 1.

(h) $\tan^{-1}\left(-\frac{1}{\sqrt{3}}\right) =$ Must be in Quad IV b/c negative, need $\sin x = \pm\frac{1}{2}$ and $\cos x = \pm\frac{\sqrt{3}}{2}$

$\Rightarrow \boxed{x = -30° \text{ or } -\frac{\pi}{6}}$

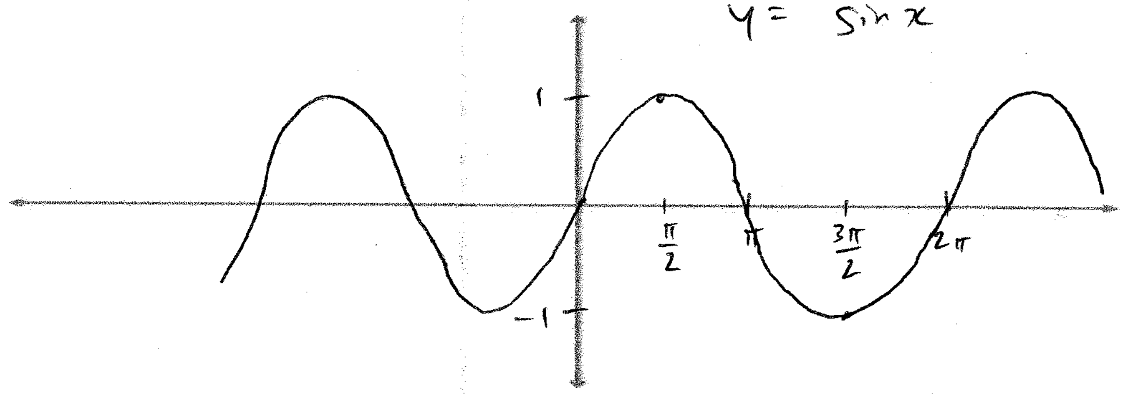


Positive:
Negative:

Positive:
Negative:

9. (**6** points) Graph $y = 4\sin(2x)$. Explain how you transform the graph of $\sin x$ to get your result (or show it by doing successive graphs), and label important points on your graph (y-values, x-intercepts, etc).

$y = \sin x$



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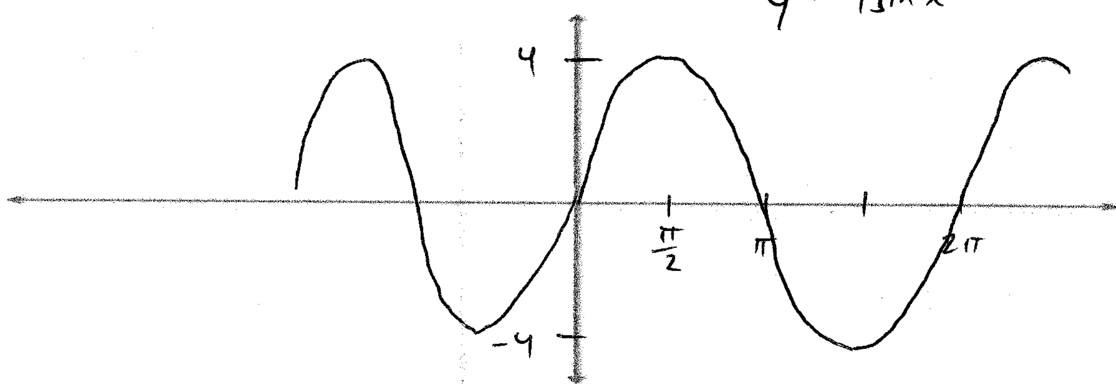
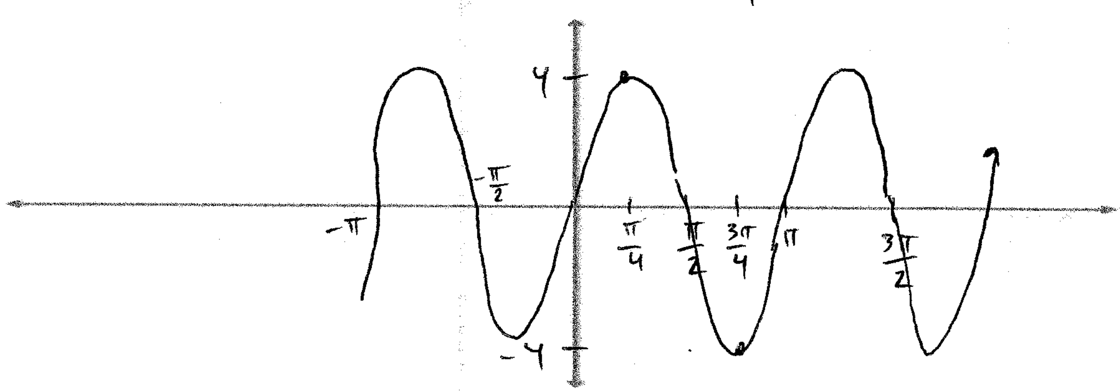
$y = 4\sin x$



2

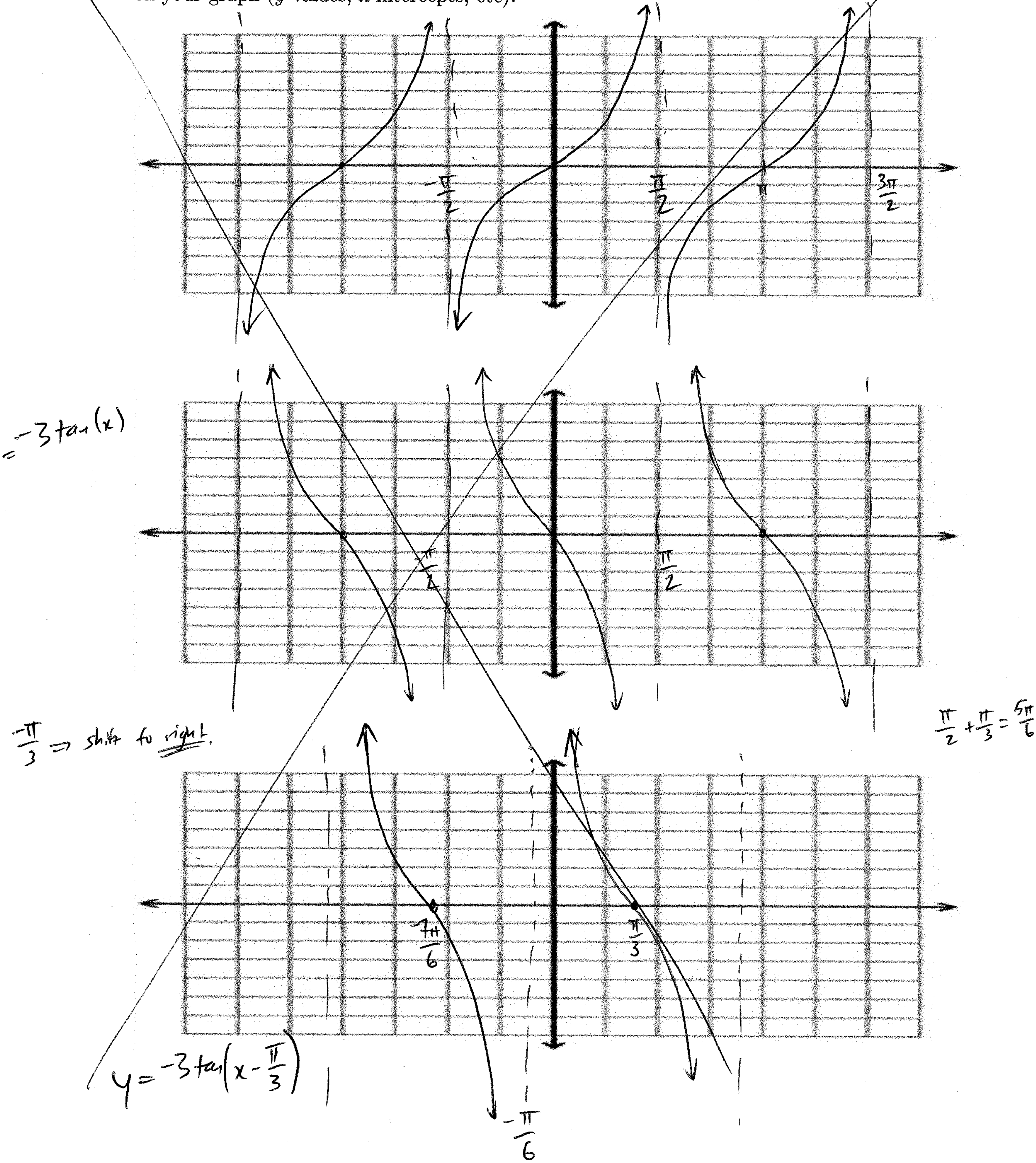
$y = 4\sin(2x)$



2

-1 for not explaining/showing each step it wrong.

10. (5 points) Graph $y = -3\tan\left(x - \frac{\pi}{3}\right)$. Explain how you transform the graph of $\tan x$ to get your result (or show it by doing successive graphs), and label important points on your graph (y-values, x-intercepts, etc).



Ex. Cred.

~~11. (5 points)~~ One of the following equations is true for all values of x in the domain, and the other equation is only true for certain values of x in the domain:

$$\sin(\sin^{-1} x) = x \qquad\qquad \sin^{-1}(\sin x) = x$$

(a) Which equation is true for all values of x in the domain, and which is only true for certain values of x in the domain? Explain how you know which is which.

(b) For each equation, give an example of a value of x that either is undefined or makes the equation false. Explain why your example x-value does not work.

(a) • $\sin(\sin^{-1} x) = x$ is true for all values in the domain of $x = -1$ to $x = 1$ b/c the sin and $\sin^{-1}$ will cancel out

• $\sin^{-1}(\sin x) = x$ is only true for x-values between $\frac{-\pi}{2}$ and $\frac{\pi}{2}$, b/c $\sin^{-1}$ will only produce those values.

(b) $\sin(\sin^{-1} 2)$ is undefined b/c the domain of $\sin^{-1}$ is $-1 \le x \le 1$ or $[-1, 1]$.

$$\sin^{-1}\left(\sin \frac{3\pi}{4}\right) = \sin^{-1}\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$

and $\frac{\pi}{4} \neq \frac{3\pi}{4}$.

$$\begin{array}{r} 9:52 \\ -9:33 \\ \hline 19 \end{array}$$